Thick-wall Cylinders:

Let:

r_i = inside radius of a cylinder

r_o = outside radius

p_i = internal pressure

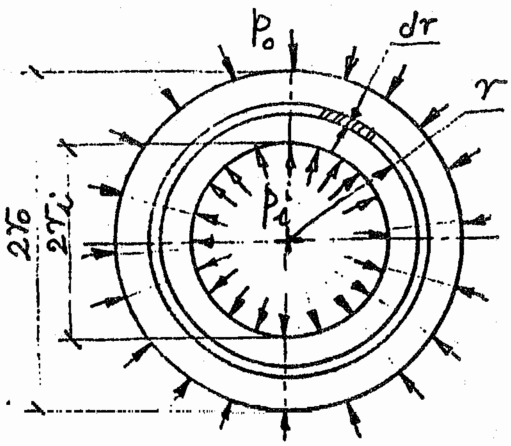
p_o = external pressure

σ_t = tangential stress

σ_r = radial stress

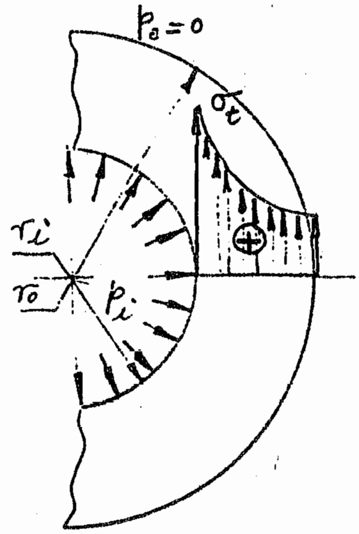
σ_l = longitudinal stress

(Principle stresses)



Then:

$$\sigma_t = \frac{p_i r_i^2 - p_o r_o^2 - r_i^2 r_o^2 (p_o - p_i)/r^2}{r_o^2 - r_i^2}$$



$$\sigma_r = \frac{p_i r_i^2 - p_o r_o^2 + r_i^2 r_o^2 (p_o - p_i)/r^2}{r_o^2 - r_i^2}$$

+ve values indicate tension

−ve values indicate compression

Also:

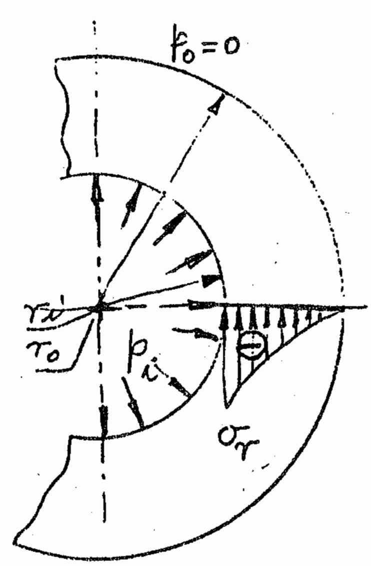
For $p_o = 0$

$$\sigma_t = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)$$

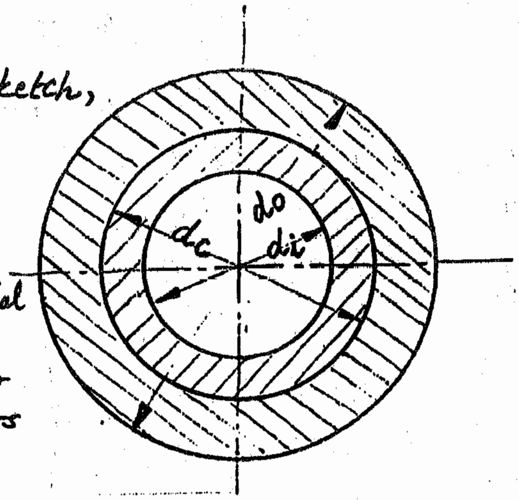
$$\sigma_r = \frac{r_i^2 p_i}{r_o^2 - r_i^2}\left(1 - \frac{r_o^2}{r^2}\right)$$

When the cylinder has closed ends:

$$\sigma_l = \frac{p_i r_i^2}{r_o^2 - r_i^2}$$



Interference Fits :



Let:

d_i , d_o , d_c are as shown in sketch,

p = pressure at contact surface.

δ = the total interference.

μ_i , μ_o = Poisson's ratio for inner and outer cylinders material respectively.

E_i , E_o = Modulus of elasticity for inner and outer cylinders material respectively.

Then:

$$p_c = \frac{\delta}{d_c\left[\frac{d_c^2+d_i^2}{E_i(d_c^2-d_i^2)} + \frac{d_o^2+d_c^2}{E_o(d_o^2-d_c^2)} - \frac{\mu_i}{E_i} + \frac{\mu_o}{E_o}\right]} \quad \text{——— (1)}$$

cases :

(1) $E_i = E_o = E$

$$p_c = \frac{\delta E}{\left[\frac{2 d_c^3 (d_o^2 - d_i^2)}{(d_c^2 - d_i^2)(d_o^2 - d_c^2)}\right]}$$

(2) $d_i = 0$, i.e. solid inner cyl. , $E_i = E_o = E$

$$p_c = \frac{\delta E}{\left[\frac{2 d_c\, d_o^2}{d_o^2 - d_c^2}\right]}$$

e.g. inner race of a ba rolling bearing on a solid shaft , a coupling hub or gear on a solid shaft, ..

(3) $d_o = \infty$, $E_i = E_o = E$

$$p_c = \frac{\delta E}{\left[\frac{2 d_c^3}{(d_c^2 - d_i^2)}\right]}$$

e.g. outer race of a ba rolling bearing in a housi a bearing bush or a ge nut of a power screw, -

Note: if the materials are different, which is the general case, use equation (1) by substituting $d_i = 0$ for case (2) and deviding by d_o^2 and getting the limit value in case (3)

Force fitting:

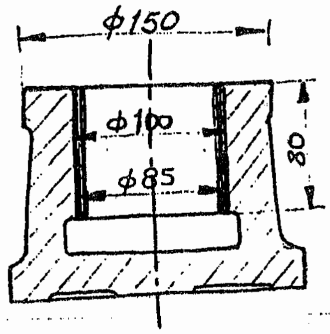
Calculate the force required to drive a Tin-Bronz bush into the CI. foot pedestal of a jib crane if:

$\mu_i = 0.3$, $E_i = 93*10^3$ N/mm^2 for G-Cu Sn 12 and

$\mu_o = 0.27$, $E_o = 80*10^3$ N/mm^2 for C.I. GG.22

The type of fit is H_8/n_7 OR H_7/n_6

μ st/CI = μ Br/CI = 0.15 - 0.25 friction coeff.



ϕ 100 H_8/n_7

Hole $100^{+0.054}_{-0.000}$, bush O.D. $100^{+0.058}_{+0.023}$

average +0.027   average +0.041

$\delta_1 = 14\ \mu$

ϕ 100 H_7/n_6

Hole $100^{+0.035}_{-0.000}$, bush O.D. $100^{+0.045}_{+0.023}$

average +0.018   average +0.034

$\delta_2 = 16\ \mu$

$d_i = 85$ mm, $d_c = 100$ mm, $d_o = 150$ mm, $l = 80$ mm

a) Contact pressure

$$p_c = \frac{\delta}{d_c\left[\frac{(d_c^2+d_i^2)}{E_i(d_c^2-d_i^2)} + \frac{d_o^2+d_c^2}{E_o(d_o^2-d_c^2)} - \frac{\mu_i}{E_i} + \frac{\mu_o}{E_o}\right]}$$

$$\frac{d_c^2+d_i^2}{E_i(d_c^2-d_i^2)} = \frac{(100)^2+(85)^2}{93*10^3(100^2-85^2)} = 0.6674*10^{-4}$$

$$\frac{d_o^2+d_c^2}{E_o(d_o^2-d_c^2)} = \frac{150^2+100^2}{80*10^3(150^2-100^2)} = 0.325*10^{-4}$$

$$\frac{\mu_i}{E_i} = \frac{0.3}{9.3*10^4} = 0.0323*10^{-4}, \quad \frac{\mu_o}{E_o} = \frac{0.27}{8*10^4} = 0.034*10^{-4}$$

$$p_{c_1} = \frac{0.014}{100}\left[\frac{10^4}{0.6674+0.325-0.0323+0.034}\right] = 1.408 \text{ N/mm}^2$$

also

$$p_{c_2} = 1.6095 \text{ N/mm}^2$$

b) Required driving force :

A_c = Contact area $= \pi(100)(80) = 8000\cdot\pi$ mm^2

Driving force $= F = p_c * A_c * \mu$

$\therefore F_1 = 1.408 * (8000\,\pi) * 0.25 * 10^{-3} = \boxed{8.85}$ kN

$F_2 = 1.6095 * (8000\,\pi) * 0.25 * 10^{-3} = \boxed{10.11}$ kN

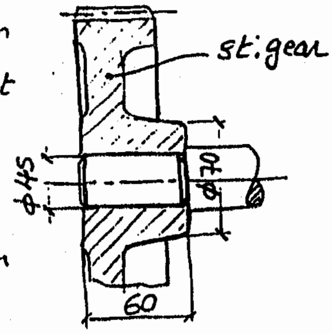
Note: Two other fits are investigated

	H_7/p_6		H_7/s_6	
Hole	$100\ {}^{+0.035}_{-0.000}$] $+0.018$		$100\ {}^{+0.035}_{\ 0.000}$] $+0.018$	
Bush	$100\ {}^{+0.059}_{+0.037}$] $+0.048$		$100\ {}^{+0.093}_{+0.071}$] $+0.082$	
average δ	0.030	mm	0.082	mm
Contact press.	3.017	N/mm^2	8.247	N/mm^2
Driving force	$\boxed{\simeq 19}$	kN	$\boxed{51.8}$	kN

The framed values of the required driving force reflect the drastic change of the force as affected by the amount of the interference employed.

Interference fit to transmit torque :

15 kW at 1450 rpm is to be transmitted from a gear-wheel hub to a 45 mm diam solid shaft as shown by the sketch. The security factor is considered 2.0 (starting under full load), the friction coefficient = 0.15 (st/st minimum dry friction coefficient).



Calculate the required interference and the type/grade of fit to allow this based on the average interference. (as the propable expected interference).

$$\text{Torque} = \frac{71620 * H.P.}{n} = \frac{71620 * 15 * 1.36 * 9.81 * 10}{1450} = 99 \quad kN.mm$$

$$\text{Design torque} = 2.0 * 99 \cong 200 \quad kN.mm$$

$$T_{design} = p_c(\pi d_c)\,\ell_c * \frac{d_c}{2} * \mu$$

$$\therefore 200,000 = p_c(\pi * 45)(60)\left(\frac{45}{2}\right)(0.15) \text{ , } p_c = \text{Contact pressure}$$

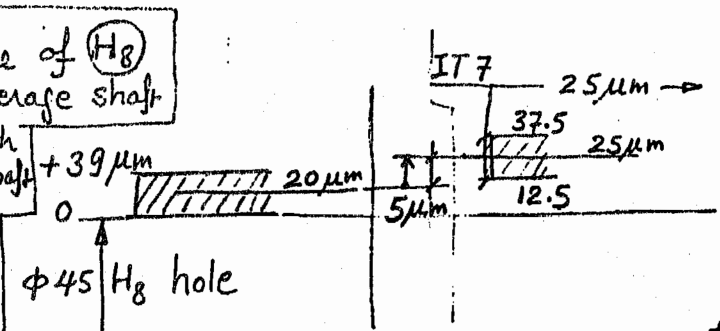
$$\therefore p_c \cong 7.0 \quad N/mm^2$$

$$p_c\left[\frac{2 d_c\, d_o^2}{d_o^2 - d_c^2}\right] = \delta E$$

$$(7.0)\left[\frac{2 * 45 * 70^2}{70^2 - 45^2}\right] = \delta\,(2.1 * 10^5) \quad \therefore \delta = 5.11 * 10^{-3} \text{ mm}$$

$$\therefore \delta = 5 \ \mu m$$

- get average hole of H8
- add δ to get average shaft
- Apply $\frac{IT7}{2}$ on both sides of average shaft
- Look for type of shaft of grade 7 that have upper & Lower limit



The required shaft tolerance based on IT7 is $^{+37.5}_{+12.5}$ with an average of +25 μ for a diameter = 45 mm.

$$45\ m_7 \equiv 45\ ^{+34\mu}_{+9\mu} \text{ — average } \frac{43}{2} = 21.5\mu$$

$$45\ n_7 \equiv 45\ ^{+42\mu}_{+17\mu} \text{ — average } \frac{59}{2} = 29.5\mu$$

$$45\ p_7 \equiv 45\ ^{+51\mu}_{+26\mu} \text{ — average } \frac{76}{2} = 36\mu$$

$45\ n_7 \equiv 45\ ^{+0.042}_{+0.017}$ is the type and grade nearest to the required average of 25 μ that gaurantee a min. average interference of 9.5 μ > 5 μ calculated.

In such a case:

$$p_{c_1} = 7 * \frac{9.5}{5} = 13.3 \quad N/mm^2$$

$$\text{Force required for fitting} = p_c * 0.25 * \pi * 45 * 60$$
$$= 28.2 \text{ kN}$$

<u>Note</u>: Force fitting will hurt the contact surfaces. It advisable to utilize heating the gear hub (or cooling the shaft)

$\Delta l = l\ \alpha\ \Delta t$, Δl = average interference = $9.5 * 10^{-3}$ mm

l = diam = 45 mm

$\alpha = 11 * 10^{-6}$ mm/mm

$$\therefore \Delta t = \frac{9.5 * 10^{-3}}{45 * 11 * 10^{-6}} = 19\ ^\circ C$$

ambient = 35 °C , losses during fitting duration ≈ 15 °C

$\therefore$ heat to $t = 19 + 35 + 15 = 69\ ^\circ C \simeq \underline{\underline{70\ ^\circ C}}$